Size Effect in Dry Snow Slab Tensile Fracture

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Abstract

Slab avalanches release following shear fracture propagation beneath and tensile fracture through a cohesive snow slab. Most slabs fracture at a homologous temperature exceeding 0.95 and are composed of 60-80% air by volume. The strength and fracture properties of snow slabs are strongly dependent on size, loading rate, and density. We present the results of hundreds of laboratory and field tests in order to investigate the size effect on the nominal tensile strength of slabs. Given the large scatter in test data, it is difficult to determine whether a deterministic fracture mechanical or a Weibull statistical size effect theory is more appropriate using test data alone. We suggest that a physical argument is necessary to justify the appropriate scaling theory.

1 Introduction

Snow avalanches can be classified into two general types: 1, loose avalanches, which form in cohesionless surface snow; 2, slab avalanches, which release large volumes of cohesive snow after the propagation of fractures, first in shear beneath a cohesive slab and then in tension through the slab [1]. Slab avalanches typically are more destructive, and claim more lives, than loose avalanches [2], and thus are the primary concern to avalanche forecasting operations, land use planners, and winter backcountry travelers.

The highly porous nature of the snow pack and the high homologous temperature of alpine snow make it difficult to analyze snow using typical engineering concepts. For densities typical of seasonal snow slabs (100-350 kg/m^3), the volume fraction filled by solids is just 0.1-0.4. Most alpine snow exists at greater than 90% of its absolute melting temperature. These characteristics give snow important rate and temperature dependent properties [3, 4], and these properties can vary widely for a given density of snow. Additionally, even the definition of a crack in a highly porous material like snow is not as simple as in most engineering materials. For these reasons, snow has often been considered one of the most difficult earth materials to test and analyze.

1.1 Size Effect on Snow Strength

Many studies on snow have reported a size effect on strength, both in shear [5, 6] and tension [7, 8]. The combined size effect in shear and tension influences the

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fracture dimensions and thus the destructive potential of slab avalanches [2]. A size effect on strength can be explained within the framework of deterministic fracture mechanics [e.g., 9], statistics [e.g., 10], or a combination of the two [11], with important differences in the large-size asymptotic strength predictions.

Weibull [10] postulated that size effects in brittle fracture have a statistical character. The first ideas about size effects in brittle fracture for snow were put forth using this theory more than 30 years ago [7, 12]. Weibull theory describes the increasing probability of finding material defects for increasing sample volume. Both Weibull theory and deterministic fracture mechanics size effect laws contain a similar result, namely that strength decreases as sample size increases.

Fracture tests for any material will have statistical scatter associated with strength values. Therefore, it is often not trivial to conclude whether experimental data are following a Weibull size effect or a deterministic fracture mechanical size effect. This problem is compounded for alpine snow since it is a highly random material with more data scatter than for typical engineering materials. A typical coefficient of variation (CoV) for field fracture tests of snow is about 0.2 [13] whereas a typical value for concrete is about 0.04 [14, 15]. Data scatter can make it more difficult to determine which size effect law, statistical or deterministic, is followed.

The physical picture of slab tensile fracture for the Weibull approach is that the entire slab fails at once by expansion of a small imperfection with orientation perpendicular to the avalanche weak layer. The physical picture for the fracture mechanics approach is that tensile crack propagation begins in the highly stressed boundary layer near the base of the slab. The fracture then propagates upward, perpendicular to the weak layer, to reach the snow surface [1].

1.2 Weibull Statistical Size Effect Law

For historical reasons and for completeness, we consider the Weibull statistical size effect law here. The two key assumptions for applicability of Weibull's theory are [9]: 1, failure occurs (or is assumed to occur) right at initiation of macroscopic fracture; 2, there is no characteristic length scale in the material. In the context of this paper the first assumption precludes any stable crack growth before failure. The second assumption is equivalent to stating that at failure the fracture process zone and any associated stress redistribution are negligible.

Repeated attempts to estimate the Weibull modulus m for snow [7, 12, 16] have yielded very low values which suggest that Weibull size effects are large for alpine snow. However, lack of care in relation to experimental technique limits conclusions. Similitude requirements must be followed to ensure that size effects can be determined. Additionally, snow samples must be taken in rapid succession from the same layer to prevent aging and settlement effects from changing the strength.

The Weibull statistical size effect law (reviewed by [9], e.g.) relates the mean strength $\bar{\sigma}_{Nu}$ to size D via

$$\bar{\sigma}_{Nu} = \bar{\sigma}_{Nu}^{\bar{D}} \left(\frac{\bar{D}}{D}\right)^{n_d/m} \tag{1}$$

where m is the Weibull modulus, $\bar{\sigma}_{Nu}^{\bar{D}}$ is the mean strength at reference size $\bar{D}$ and n_d is the similitude dimension. For beam bending tests with a constant span to depth ratio (and constant width), n_d = 2.

From [13], the mean nominal tensile strength of snow from hundreds of in-situ uniaxial tensile tests (n_d = 1) for an average slab thickness $\bar{D}$ = 0.2 m is given by

$$\bar{\sigma}_{Nu}^{\bar{D}} = 80 \left(\frac{\rho}{\rho_i}\right)^{2.4} \tag{2}$$

where ρ is the snow density, ρ_i is the density of ice (917 kg/m^3), and $\bar{\sigma}_{Nu}$ is in kPa.

The Weibull modulus m can be obtained in a number of ways. Test data can be fit to the Weibull distribution using, for example, the maximum likelihood method or least squares. For test data involving specimens of different sizes, the slope of the size effect in a log-log plot of nominal strength versus size can be related to the power law exponent of Equation 1. Finally, m can be calculated using the CoV of strength for tests of the same size and geometry [9] using the equation

$$CoV = (0.462 + 0.783m)^{-1} \tag{3}$$

in the range $5 \leq m \leq 50$ where accuracy is within 0.25 percent. Since the CoV is independent of specimen volume m should be also. For a given set of strength data it may be possible to calculate m in more than one way. The values of m should be identical if Weibull theory is applicable [14]. For any method similitude requirements must be followed to avoid shape effects.

1.3 Fracture Mechanical Size Effect Law

The deterministic fracture mechanical size effect takes a different form depending on whether a structure fails after large stable crack growth (Type 2) or at crack initiation in a boundary layer (Type 1) [14]. The tensile fracture of cohesive snow slabs is considered a Type 1 fracture [2]. The simplest equation that expresses the Type 1 size effect for unnotched samples [14] is

$$\sigma_{Nu} = f_r = f_{r\infty} \left(1 + \frac{D_b}{D}\right), \tag{4}$$

where σ_{Nu} is the nominal strength, f_r is the modulus of rupture, $f_{r\infty}$ is the large size asymptotic limit of the modulus of rupture and D_b is interpreted as roughly twice the thickness of boundary layer cracking and is considered a material property. The characteristic specimen size D for slab avalanches is the slope-normal slab thickness.

The modulus of rupture f_r for an unnotched beam is

$$f_r = \frac{6M}{bD^2}, \tag{5}$$

where M is the applied bending moment, b is the specimen thickness and D is the beam depth [9]. For the present case of four-point bending with load applied at third points of the beam, the maximum bending moment is at peak load P giving $M = PS/6$ where S is the beam span. This leads to the following expression for the nominal strength, given $S/D = 4$ used in this study:

$$\sigma_{Nu} = \frac{PS}{bD^2} = \frac{4P}{bD}. \tag{6}$$

Combination of Equations 6 and 4 allows the material parameters $f_{r\infty}$ and D_b to be calculated from test data at different sizes D.

2 Methods

2.1 Laboratory Fracture Tests

We used a cold laboratory located at Rogers Pass in Glacier National Park of Canada for carrying out bending tests on beams of cohesive, dry snow. The temperature of the laboratory was manipulated to keep the samples at the same temperature as when they were extracted from the snow pack. The snow was collected from a study plot near the laboratory, and was limited to dry snow with no liquid water present. The bending tests were carried out using a standard tension/compression testing machine under displacement control using a four-point bend fixture.

Since the snow specimens and the range of temperatures of interest for slab avalanches are near the melting point, weight-compensation was necessary to avoid viscous deformation during sample preparation and test setup. This was achieved by placing the entire testing machine on its side and orienting the tests horizontally. The samples were supported by smooth plastic tables which straddled the test machine. The friction coefficient between the snow and the tables, important as the sample deforms during testing, can be measured easily and accounted for in the strength calculation.

2.2 In-Situ Fracture Tests

In-situ uniaxial tensile tests in snow were carried out by [8] and [13]. The slab samples were isolated in tension by inserting a smooth plastic sheet into the snow pack to act as a sliding surface. Stainless steel frames were used to grip the beam-like snow samples which had one end attached to the snow pack. A progressive force was applied until the sample fractured in tension. The friction between the snow and the sheet was measured and accounted for in the calculation of the tensile strength.

Cantilever beam tests have also been reported for snow [16]. In these tests, beam-shaped samples were extracted from the snowpack using a long stainless steel cutter. The beams were then gradually pushed out of the cutter such that the cantilever length increased until the beams failed under self weight. The slab density, temperature, and dimensions at failure were used to calculate a value of the nominal strength of the sample.

Table 1 shows the data series used for calculations in this paper, containing 137 fracture tests. In addition, we review published data for the CoV for hundreds of in-situ tensile tests [13] and for the Weibull modulus and size effect from cantilever beam tests [16].

Series	Test Type	Time to Failure (s)	Density (kg/m^3)	Size (cm) and (#)	Mean Snow Temp. (°C)	Source
A	4PB-H[1]	0.20-0.33	322 ± 12	5(12) 10(12)	-7.2	Present Study
B	4PB-H[1]	0.15-0.55	296 ± 14	5(8) 10(7) 20(5)	-6.5	Present Study
C	4PB-H[1]	0.2-0.35	270 ± 20	5(8) 10(7) 20(6)	-7.4	Present Study
D	IT[2]	1.7 ± 0.3	216 ± 8	[3](42)	-7.3	[8]
E	IT[2]	2.8 ± 0.8	251 ± 6	[4](30)	-9.4	[8]

Table 1: *Unnotched snow slab fracture data referenced in this study.* [1]*Weight-compensated four-point bending loaded at third points.* [2]*In-situ uniaxial tensile test.* [3]*Mean cross section 16 cm × 12 cm.* [4]*Mean cross section 16 cm × 20 cm.*

3 Results

Figure 1 shows Series B and C with a fit to Equation 4 using a Levenberg-Marquardt nonlinear least squares algorithm. Values of σ_N were calculated using Equation 6. The fitting algorithm provides an estimate of the covariance matrix for calculating the standard error of the model parameters.

Figure 2 shows a log-log plot of nominal strength versus specimen size for Series A, B, and C. Series A included only two different specimen sizes. The slope of the

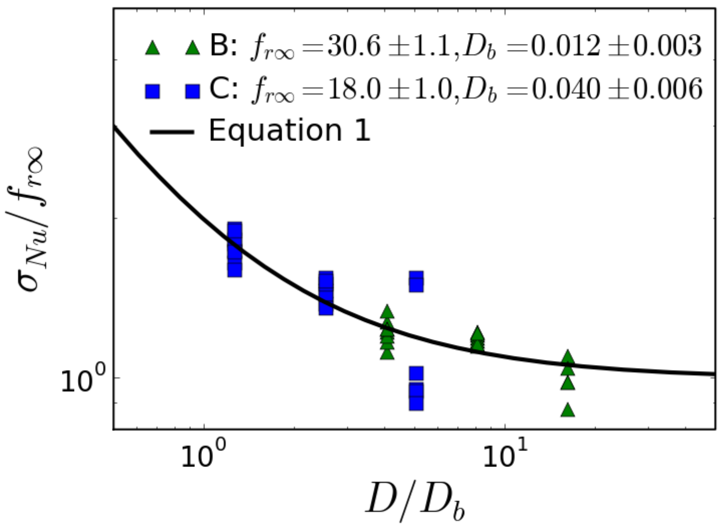

Figure 1: *Nonlinear fit of fracture mechanical scaling Equation 4 to Series B and C. Parameter values from fit shown in legend,* $f_{r\infty}$ *in kPa and* D_b *in m.*

fit for Series A is not statistically significant, reflecting some combination of the limited range of sample sizes and the smaller maximum size relative to Series B and C.

The slope of the fits to Series B and C in Figure 2 were used to calculate values for the Weibull modulus m which are shown in Table 2. We determined m for Series D by fitting the published data to a Weibull distribution using least squares with no strength threshold parameter, with 95% variance explained. We also determined that the data follow a normal distribution more closely, with 98% variance explained, as suggested by [8].

m	Series	Calculation Method
14	B	slope of measured size effect (Fig. 2)
6	C	slope of measured size effect (Fig. 2)
15	A, B, C	average CoV for 3 sizes (Eq. 3)
5.8 $\pm$ 0.5	D	least-squares fit to Weibull distribution
5.8	D, E	reported CoV in [13] (Eq. 3)
5.2	*	average CoV for 33 test series [8, 13] (Eq. 3)
1.5 $\pm$ 0.5	F	reported in [16]

Table 2: *Weibull modulus calculated in different ways from the data series in this paper. The CoV is related to the Weibull modulus via Equation 3. *Series D and E plus field data not shown in Table 1.*

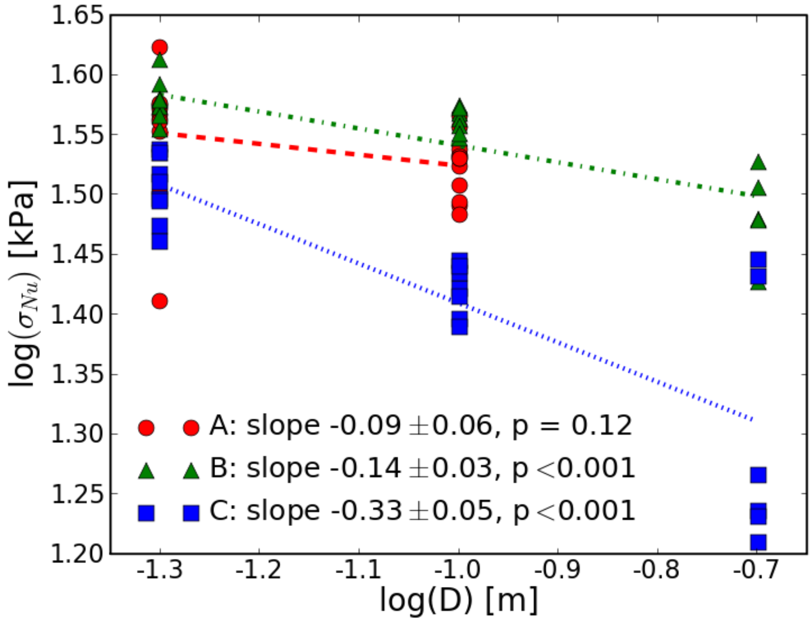

Figure 2: *Nominal strength versus size for Series A, B, and C. The dotted lines are linear least-squares fits to each individual data set. The slope and p-value of each fit are shown in the legend.*

For both Series D and E, the CoV was reported as 0.2 [8, 13]. The value of the CoV averaged over 33 snow layers with hundreds of samples is 0.22 [13]. Series A, B, and C contained similar snow which was sampled from the same layer within the snow pack on three consecutive days. The CoV for each of the three sample sizes for all three series was averaged (CoV $\approx$ 0.08) and also used to calculate m from Equation 3. The number of samples for each individual size was too small to permit a separate comparison of the CoV for each size.

Combining Equations 1 and 2 leads to a Weibull size effect law of

$$\bar{\sigma}_{Nu}^{\bar{D}} = 80 \left(\frac{\rho}{\rho_i}\right)^{2.4} \left(\frac{\bar{D}}{D}\right)^{n_d/m} . \tag{7}$$

For one-dimensional similitude and a size increase from 0.2 to 1 m, Equation 7 predicts a decrease in strength of 10% to 66% given the range in values of m in Table 2. Given the respective values for D_b for Series B and C (Figure 1), the Bažant size effect law (Equation 4) predicts a strength decrease in the range 5% to 13% for the same size increase. Any controlled tests on sizes approaching 1 m would be extremely difficult to carry out for snow in order to test the predictions of the competing size effect laws here.

The difference between the values of m for Series B and C is large given the similarities in the material properties. The series were sampled from the same layer within the snow pack on two consecutive days and contained snow of very similar temperature, hardness, and grain size and type. The biggest difference was in the density, and this difference was primarily limited to the large samples in Series C. The lowest density samples in Series C were four out of the six large sizes; the remaining samples were very near the density of Series B. This most likely reflects a fundamental and unavoidable difficulty in extracting large homogeneous samples from a highly variable and random snow pack. If not for these low density (and lower strength) samples, the slope of the size effect for Series C may have been closer to that of Series B. This would effectively increase m toward the values calculated for Series B ($m = 14$) and based on the CoV for all data in Series A, B, and C ($m = 15$). This would also make it more difficult to rule out Weibull theory on the basis of conflicting values of the Weibull modulus for different calculation methods.

The wide range in values calculated and reported for the Weibull modulus in Table 2 can be explained partly on the basis of differences in experimental methods and snow properties. However, it remains doubtful whether the range could be narrowed much more. The value of $m \approx 6$ from Series D and E is likely near the maximum that can be expected from field tests. The data scatter inherent in a highly porous and random structure such as a snow pack combined with the large number of variables that cannot be controlled in a field test limit the minimum value that can be expected for the CoV. The averaged CoV from the laboratory tests is about 2.5 times lower than the field tests and leads to a corresponding increase in m.

The cantilever beam tests of [16] were on the small end of the range of sizes considered in this paper (10 cm and less). No size effect was observed in the data, which is surprising given the very low value of m. However, the cantilever beam tests did not satisfy similitude, so size effects cannot be easily measured. Additionally, the results [16] show poor fits to the Weibull distribution. Their predicted tensile strength for a density of 250 kg/m^3 is 10 kPa which is almost 300% higher than the results of the careful in-situ experiments of [13].

Series A also contained a maximum specimen size of 10 cm, and the slope of the linear regression for this series in Figure 2 was not statistically significant at the 95% level. It is possible that samples larger than 10 cm may be necessary in order to begin to observe a size effect in the strength of snow. This suggests the presence of a length scale that would violate one of the fundamental assumptions of Weibull theory.

For the snow slab, it is not possible that fracture initiates from small size imperfections (mm to cm scale) since a typical slab (or weak layer) contains millions of them. If such were true, slab avalanches would be much more common than they are. In fact, it would be impossible for alpine snow to exist on mountain slopes if fracture initiated from small scale imperfections on the order of the grain size. In addition to precluding Weibull theory, this suggests that Linear Elastic Fracture Mechanics (LEFM) is not appropriate.

For shear fracture initiation in the weak layer beneath the slab [17] and [18] estimate that the FPZ is on the order of at least several cm and is a significant fraction of slab thickness D. In tension, the material property D_b can be related to the size of the fracture process zone [14]. The values for D_b calculated in this paper ($\sim$1 and 4 cm) are between 10 and 100 times the average grain size (0.5-1.0 mm grains in Series B and C). The evidence of a macroscopic length scale favors a fracture mechanical explanation of the size effect.

Both of the important assumptions for application of Weibull statistics might be satisfied for extremely large avalanche sizes where failure (a Type 1 size effect) initiates from near the bottom of the slab as suggested by [2]. For very large sizes ($D >> D_b$) and Type 1 tensile failures, a non-local Weibull theory may apply with asymptotic form $\sigma_{Nu} = C_1 D^{-n_d/m}$ [14, 11]. In this case, full section failure could happen right at the onset of macroscopic fracture. Verification would be difficult given the challenges for sampling large sizes discussed above. Even higher data scatter would be expected compared to small-scale tests of the type presented here.

Any experimental results will be characterized by data scatter, and some statistical evaluation will be necessary to measure a size effect. The choice of an appropriate theory will have to be made on the basis of the most realistic physical model. For concrete, the Weibull modulus is approximately 24 [15] and it can be argued that the Bažant size effect law is a better fit to concrete data. However, for alpine snow, with more data scatter and a lower Weibull modulus, this line of reasoning is less conclusive.

5 Conclusions

The basic assumptions of the Weibull statistical size effect and the Bažant fracture mechanical size effect were applied to explain the size effect on the tensile strength of snow slabs. Data from laboratory bending tests, in-situ uniaxial tensile tests and cantilever beam tests were used to calculate respective parameters for both theories.

The assumptions necessary to apply Weibull theory are probably not fulfilled except for extremely large avalanches. However, the large scatter in snow, and thus low Weibull modulus m, make a conclusive assessment of the applicability of Weibull theory difficult on the basis of data alone. A deterministic fracture mechanical explanation of the size effect offers a better theoretical basis and agrees with numerous estimates of a macroscopic fracture process zone for snow.

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