Extreme avalanche runout in space and time

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Abstract: The distribution of avalanche runout varies in space and time for individual avalanche paths and from mountain range to mountain range. In this paper, such variations are considered based on the assumption (supported by data worldwide) that the spatial distribution of extreme avalanche runout follows a Gumbel distribution and that the arrival rate of avalanches can be modelled as a Poisson process. The input required is a set of extreme avalanche runout distances for the mountain range and a knowledge of avalanche frequency at the beginning of the runout zone for the path in question. Such information allows theoretical estimation of the effective return period as a function of position, which is very important in zoning applications. In addition, general expressions are derived to relate Gumbel parameters for different mountain ranges to a frequency index to explore general frequency implications from one mountain range to another. The estimated Gumbel parameters imply consistent relationships for expected avalanche frequency and terrain from one mountain range to another.

Key words: snow avalanches, runout distances, return period, frequency, terrain.

Résumé : La distribution des parcours d'avalanches varie dans l'espace et dans le temps tant pour les cheminements individuels des avalanches que d'une chaîne de montagnes à une autre. Dans cet article, l'on considère de telles variations en partant de l'hypothèse (appuyée par des données à travers le monde) que la distribution spatiale des parcours extrêmes d'avalanches suit la distribution de Gumbel et que le taux d'arrivée des avalanches peut être modélisé comme un processus de Poisson. Les entrées requises est un ensemble de distances extrêmes de parcours d'avalanches pour la chaîne de montagnes et une connaissance de la fréquence d'avalanches au début de la zone de parcours pour le cheminement en question. De telles informations permettent une estimation théorique de la période de récurrence possible en fonction de la position qui est très importante dans les applications du principe de zonage. De plus, l'on dérive des expressions générales pour mettre en relation les paramètres de Gumbel pour différentes chaînes de montagnes, à un indice de fréquence donné pour explorer les implications générales des fréquences d'une chaîne de montagnes à une autre. Les paramètres estimés de Gumbel impliquent des relations consistantes pour la fréquence d'avalanches et le terrain prévus d'une chaîne de montagne à une autre.

Mots clés : avalanches de neige, distances de parcours, période de récurrence, fréquence, terrain.

[Traduit par la Rédaction]

Introduction

Specification of expected runout distances and return periods in the runout zone is the first and most important step for zoning in snow avalanche prone terrain. The traditional method since the work of Voellmy (1955) has been to select friction coefficients in a dynamics model in a deterministic sense or with return periods attached to input friction coefficients related to expected fracture heights (e.g., Salm 1993, 1997). However, the work of Bovis and Mears (1976) and Lied and Bakkehøi (1980) introduced an entirely new method. These researchers found that extreme runout (variable return periods of 50–300 years) can be predicted from topographic terrain parameters for a set of avalanche paths in a mountain range from probability and statistics with uncertainty quantifiable statistically. The key result of Lied and Bakkehøi is the definition of a reference point from which to measure runout distances (called the β point). This point is shown in Fig. 1 as the position along the incline where the slope angle first declines to 10° proceeding downslope from the start position. This statistical method (including the reference position) is now used in many countries and jurisdictions in the world but the dynamics method is still very popular.

One criticism of the topographic (or statistical) method (B. Salm, personal communication, 1996) is that there is no obvious way to specify expected return period increases in the runout zone as the runout distance increases. The primary focus of the present paper is to remove this criticism by considering runout to vary in space and time in the runout zone. The avalanches are assumed to arrive according to a Poisson distribution with a known or estimated return period upslope of the runout zone. The distribution of runout distances in space is assumed to follow a Gumbel distribution, as shown by McClung and Mears (1991), with parameters estimated from the historical record of extreme runout in the mountain range in question. The combination of these two assumptions results in a theoretical prediction of effective return period in the runout zone for direct input into zoning calculations.

The previous discussion refers to procedures for a specific avalanche path. However, the variation of avalanche runout in space and time should also be related to the terrain and

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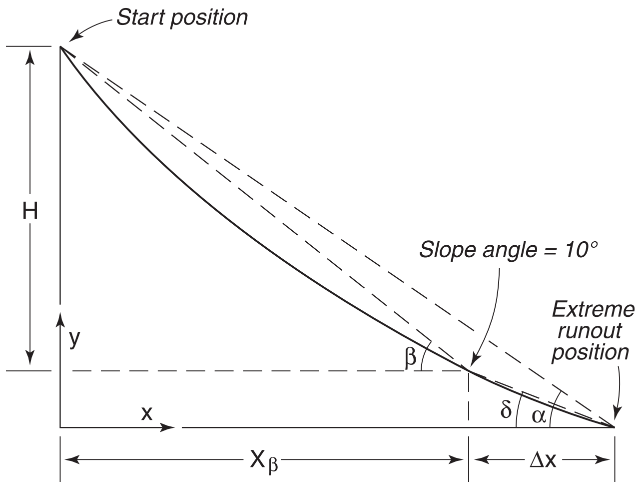
Fig. 1. Geometry for regression and runout ratio (RR) calculations, where RR = $\Delta x/X_\beta$. α, angle sighting from extreme runout position (return period 50–300 years) to start position angle; β, angle sighting from position where the slope angle first declines to 10° proceeding downslope to the start position; δ, angle sighting from α to β positions; H, vertical drop to β position.



general snow climate of the mountain range. McClung and Mears (1991) considered the effect of general climate from one mountain range to another but they did not relate runout distances to climate and expected avalanche frequency. In this paper, the general avalanche frequency is related to extreme-value (Gumbel) distributions estimated for eight mountain ranges. The analysis indicates a consistent relation between Gumbel parameters from one mountain range to another and it appears possible to relate general avalanche frequency in a mountain range to the historical record of extreme runout for that range.

The paper contains two important new results: prediction of effective return period in the runout zone for individual avalanche paths from a statistical method, and the relation of expected avalanche frequency and terrain to extreme avalanche runout data for different mountain ranges. Both results are achieved by assuming that avalanche events arrive in a temporal sense according to a Poisson process (Smith and McClung 1997).

Return period in the runout zone for individual avalanche paths

Consider first a method for predicting return period spatially in the runout zone based on a knowledge of empirical probabilistic relations for extreme runout as a function of position. The problem simply stated is as follows: given a flux of independent, discrete events which arrive at the runout zone with an estimated annual frequency (mostly determined by snow supply) and given a probabilistic determination of runout distance downslope from the position of the frequency estimate, what is the expected return period of events as a function of position in the runout zone?

The analysis in this paper provides a mathematical formulation of the problem. The two inputs required are: (1) an empirical relation for the cumulative distribution function (CDF) for extreme runout spatially, based on the historical record from a given mountain range; and (2) an estimate of avalanche frequency (or return period) at some reference point (initial value) upslope of the runout zone. Given these inputs in combination, empirical estimates are provided of return period into the runout zone increasing from the initial value (at the reference point) according to the probability distribution of extreme runout in space.

Specific avalanche paths: extreme runout and return period variations in the runout zone

This section contains the analysis for specifying avalanche runout in space and time for individual avalanche paths given that avalanche runout obeys a Gumbel distribution (or another appropriate distribution) determined from the historical record for the mountain range in question and that avalanches arrive according to a Poisson distribution in the runout zone, where the arrival rate may be found from records or vegetation damage or some other means at some point along the path. Given these inputs, a simple model can be produced from the fundamental theorem of extremes to predict exceedance probability (or return period) as a function of position in the runout zone. Keylock et al. (1999) have also made spatial predictions for return period as a function of position on avalanche paths by developing a simulation model using Monte Carlo techniques based on avalanche runout data.

(1) *Temporal variation*: Avalanche occurrences may be thought of in first approximation as random, discrete, rare independent events. Föhn (1975, 1978) and Smith and McClung (1997) showed to a good approximation that the temporal arrival of avalanche events can be modelled as a Poisson process. For a Poisson process, the *actual* arrival rate of events is a random number, n, and the *expected* number of events is given by $\mu_0 = L/T_0$, where μ_0 is the Poisson parameter, T_0 is the return period in years at a reference position near the beginning of the runout zone, and L is the time exposed, taken numerically as 1 year ($\mu_0 = 1/T_0$) to represent annual arrival rate. The probability mass function P_n is given by

$$[1] \qquad P_n(\mu_0, n) = \frac{\exp(-\mu_0)\mu_0^n}{n!}$$

(2) *Spatial variation*: The spatial cumulative distribution function (CDF) of extreme runout is represented as $F_X(x)$ for a Gumbel distribution of extreme runout distances determined for a mountain range, where X represents the variable RR = $\Delta x/X_\beta$, and x represents a particular value of RR. The runout ratio RR = $\Delta x/X_\beta$, where Δx is the runout distance (horizontal reach to the runout position from the β point), and X_β is the horizontal reach from the start position to the β point (Fig. 1). See Appendix 1 for a definition of $\Delta x/X_\beta$ in terms of angles and the slope profile for an avalanche path. McClung and Mears (1991) showed that $F_X(x)$ is adequately represented by a Gumbel distribution of runout ratio X for data from five different mountain ranges representing more than 500 avalanche paths, and further evidence is provided in this paper (see fits in Table 1) with data from three more ranges. A brief description of a method for determining the

Table 1. Frequency index n and spread parameter a for the exponential Poisson crossing model.

Mountain range	n	a	Remarks	R^2	N
Sierra Nevada	6.5	0.20	Maritime	0.98	90
Alaska	5.5	0.11	Maritime	0.97	52
Norway	4.4	0.09	Maritime	0.98	127
Northwest Iceland	4.2	0.13	Maritime	0.94	25
Colorado	4.2	0.20	Continental	0.98	130
British Columbia coast	3.4	0.09	Maritime	0.97	31
Canadian Rockies	2.5	0.08	Continental	0.96	125
Eastern Iceland	1.7	0.13	Low frequency[a]	0.96	20

Note: Mean frequency by regression 4.3. R^2, squared correlation coefficient for the fit of runout ratios to the Gumbel distribution; N, number of paths. The parameters n and a are from eqs. [15] and [16] evaluated by least squares fit to the Gumbel distributions (Appendix 1).

[a]Eastern Iceland is expected to have a low frequency of avalanching.

CDF with the geometry defined as in Fig. 1 is provided in Appendix 1.

(3) *Extremal temporal and spatial variations*: By the fundamental model of extremes (see, e.g., Benjamin and Cornell 1970, p. 271), if a random number n of extreme avalanche events occur per year and it is assumed that they all obey the same parent CDF, then the extremal, spatial CDF for extreme runout is given by $[F_X(x)]^n$. A similar assumption that all runout distances obey the same parent CDF is implicitly made when topographical models are used to specify runout in applications.

Now consider the variable $V = V(x, \mu_0)$, which describes the temporal and spatial CDF of extreme runout at position x: $F_V(v)$ with a Poisson arrival of events entering the runout zone. The extremal CDF of V at position x is given by the compound distribution (e.g., Benjamin and Cornell 1970, p. 307; Maes 1986)

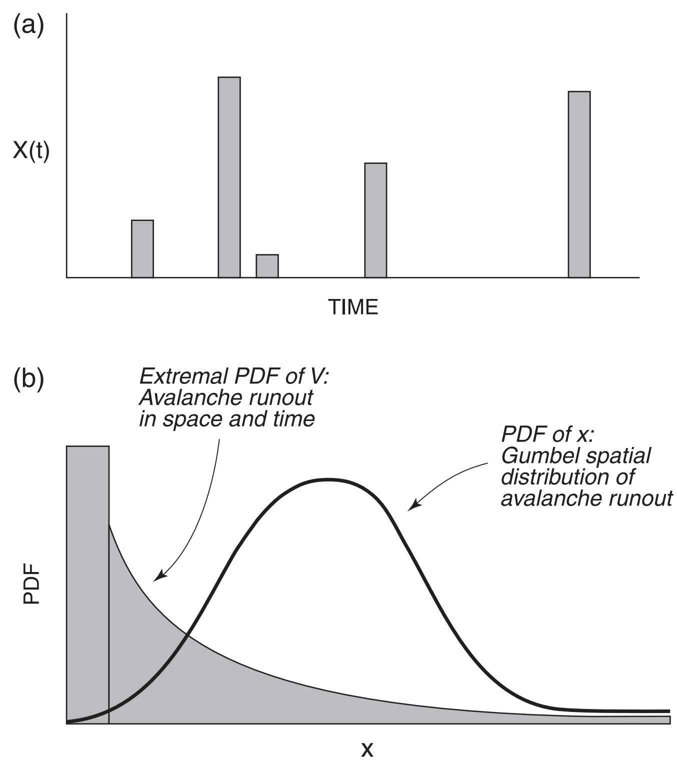
$$[2] \qquad F_V(v) = \sum_{n=0}^{\infty} P_n(\mu_0, n)[F_X(x)]^n = \exp[-\mu_0(1 - F_X(x))]$$

This important relation combines the spatial distribution of extreme runout for a mountain range (mostly dependent on terrain) with the temporal expectation of extreme annual avalanche arrival at a reference location: $\mu_0 = 1/T_0$ (mostly dependent on snow supply; Smith and McClung 1997). The exceedance probability is then given by $P_e(v) = 1 - F_V(v)$, and the effective return period as a function of position x is $T(x, \mu_0) = 1/P_e(v) = 1/[1 - F_V(v)]$. Figure 2 shows schematics of the probability density functions of extremal distributions and $X(t)$ following Maes (1986), where t is time.

Calculation of the return period (or exceedance probability) requires (1) an analysis of extreme runout for a set of avalanche paths in a mountain range to input information about the historical record of extreme avalanche runout in the range in question (e.g., the CDF of a Gumbel distribution of runout ratios (RR) or another suitable CDF), and (2) a knowledge of the expected arrival rate (or return period, T_0) of extreme avalanches for the avalanche path in question. A reference position must be selected for extreme avalanche arrival rate, for example, return period at the β point. No further statistical information is needed and the use of more extensive extreme value analysis is avoided.

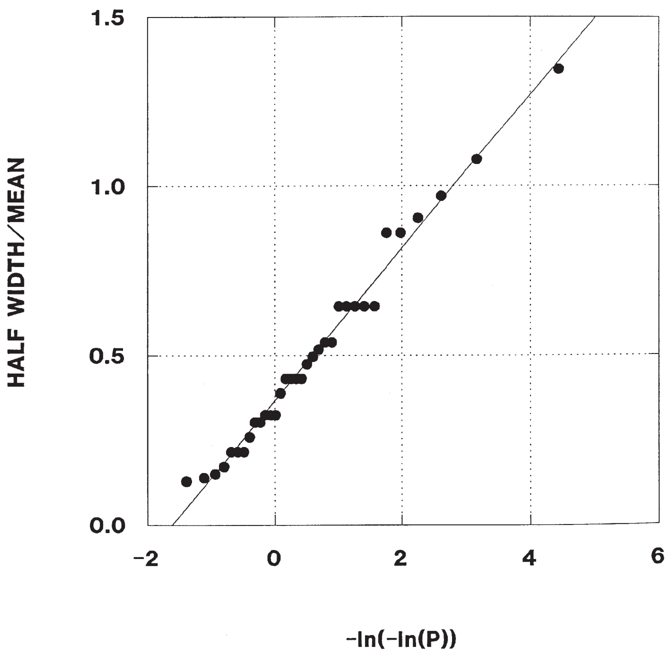
In practice, eq. [2] is applied for positions downslope from the reference position chosen for the arrival rate, and

Fig. 2. (*a*) Discrete avalanche events spaced randomly in time and runout magnitude (X). (*b*) Probability density functions (PDF) for avalanche runout (X) and V, runout in space (x) and time (μ_0). After Maes (1986). The bar on the left side of the PDF indicates the contribution for $n = 0$ in eq. [2], the probability of no occurrence.



$F_X(x)$ represents the conditional cumulative distribution of events given that they reach or exceed the reference position. At the reference position, the return period is T_0. As written, eq. [2] supplies this condition for the initial value (lowest value) of x in $F_X(x)$. The material in Appendices 1 and 2 and the theoretical development later in this paper contain information to enable eq. [2] to be used with any reference position if $F_X(x)$ is chosen as a Gumbel distribution or an exponential distribution. For positions downslope from the reference position, the return period increases according to eq. [2] as $F_X(x)$ increases with an increase in x.

Fig. 3. Runout zone width for unconfined avalanche paths fitted to a Gumbel distribution for data from Iceland. The ordinate is the half width divided by the mean of the data (232 m); R^2 = 0.99 and standard error = 0.035. The abscissa is the reduced variate –ln(–ln(P)), where P is the nonexceedance probability for the distribution: 1 – exceedance probability.



For example, if the spatial exceedance probability for avalanche runout at a location is 0.1 (F_X = 0.9) (the 1:10 runout) and if the return period at a reference position upslope is 10 years, the effective return period at the location is about 100 years for direct input into risk calculations.

The choice of reference position will affect the errors in return period determination. An example calculation for an avalanche path with long return period (Lied et al. 1998) is presented in Appendix 2 for two different choices of reference position. In practice, it is usually possible to find a position upslope of the runout zone where avalanche frequency is high enough to be estimated from observations or the methods described by McClung and Schaerer (1993). This makes the method very practical.

Model for extreme width

In avalanche land-use planning, often information is sought about the width across the slope perpendicular to the centreline of the avalanche path and the probability of avalanche debris striking facilities off the centreline of the path. When the runout zone is channelled or with gullies such terrain features can determine the width. However, often the runout zone is unconfined (not channelled or with gullies) and statistical methods may be of use to determine boundaries across slope. Data from Iceland show that extreme width of avalanches in the runout zone obeys extreme-value statistics. Figure 3 shows extreme-width data from 34 avalanche paths in Iceland fitted to a Gumbel distribution: the variable used is the distance from the centreline divided by the mean width of the Icelandic data (232 m). A CDF for coordinate w is defined from Fig. 3 starting from 0 at the avalanche path centreline. The function $F_W(w)$ then represents the CDF of coordinate distance for extreme avalanche width starting at the centreline and the value of the exceedance probability at position w is given by $1 - F_W(w)$. Assuming $F_W(w)$ and $F_X(x)$ are statistically independent gives the CDF as a function of x, w, and μ_0:

$$[3] \qquad F_Z(z) = \exp[-\mu_0(1 - F_X(x))(1 - F_W(w))]$$

where $Z = Z(\mu_0, x, w)$. The width prediction contained in eq. [3] contains only one extreme width applied to the entire runout zone. A more sophisticated width model with increase in width downslope into the runout zone is given by Keylock et al. (1999). There are not yet enough data to check the assumption of statistical independence in eq. [3].

From eq. [3] the exceedance probability may be calculated at any spatial position in the runout zone (e.g., past the β point) from the historical record of extreme runout and width for avalanches in the mountain range and the return period at the β point (or another reference): $\mu_0 = 1/T_0$. The return period in the runout zone is then

$$[4] \qquad T(z) = T(x, w, \mu_0) = \frac{1}{1 - F_Z(z)}$$

Encounter probability

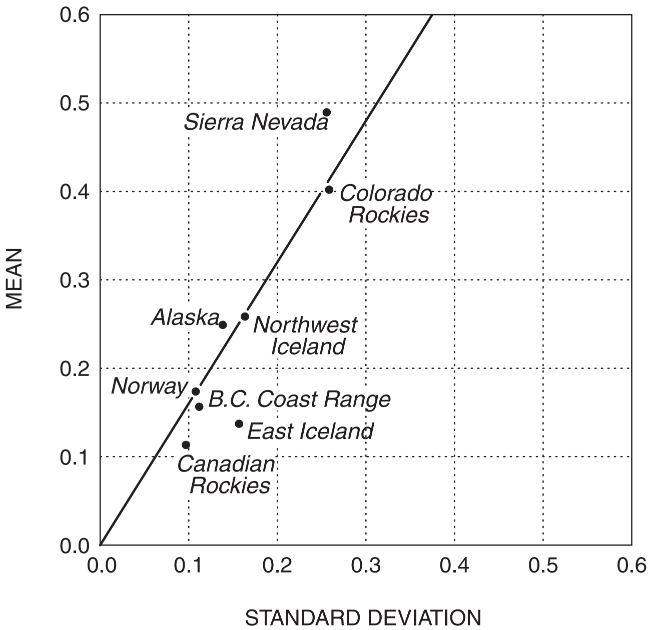
The encounter probability is of central importance in zoning applications in combination with return-period estimates. It is defined (e.g., McClung 1999) as the chance of avalanches reaching a spatial position *x at least once* during a time period L with a return period T. For eq. [1], the Poisson parameter μ is represented by $\mu(x, w, \mu_0) = L/T(x, w, \mu_0)$ and the encounter probability E_p is the sum of all terms of eq. [1] except the first (n = 0) to give

$$[5] \qquad E_p(x, w, \mu_0) = 1 - \exp\left[\frac{-L}{T(x, w, \mu_0)}\right]$$

For example, if w = 0 (the centreline of an avalanche path), the return period of avalanches reaching the runout zone is 10 years (μ_0 = 0.1), F_X at a position is 0.9 (the 1:10 extreme runout avalanche path), and L = 1 year, then the encounter probability at the location is 0.01 (a value close to $1/T(x, 0, \mu_0)$). If μ_0 = 1, L = 1 year, and F_X = 0.7 (the 1:3.33 avalanche runout), then the encounter probability is 0.283 (1:3.52).

When the ratio $L/T(x, w, \mu_0)$ is <<1, the encounter probability is equivalent to $L/T(x, w, \mu_0)$, and if L = 1 year, then the annual encounter probability is equivalent to the reciprocal of the return period (or the exceedance probability) at the location. In applications L is often taken as the useful lifetime of facilities or exposure of forest cover in avalanche terrain. If L = 100 years and $T(x, w, \mu_0)$ = 100 years, then E_p = 0.63. Thus, if the return period is 100 years at a location, there is a 63% chance of observing at least one event with 100 years of observations at the location.

Fig. 4. Mean ($u + \gamma b$) of Gumbel distributions versus standard deviation $[\pi/(6)^{1/2}]b$. The regression line has $R^2 = 0.86$ (where R is the correlation coefficient) with a slope 1.58. The constant (intercept) for the line is not significant.



Frequency and terrain interpretation of Gumbel parameters for different mountain ranges

The time and spatial distribution of avalanche runout for Gumbel parameters from one mountain range to another is investigated in this section. The modelling concepts are related to the section Return period in the runout zone for individual avalanche paths.

Data on extreme runout from different mountain ranges show that there is a consistent relation between the scale and location parameters u and b, respectively (see Appendix 1 for definitions of u and b): as the Gumbel (or type I extreme-value distribution) location parameter increases so does the Gumbel scale parameter. Figure 4 shows a plot of the mean value $\mu_G = u + \gamma b$ (where γ is Euler's constant or 0.57721) versus the standard deviation $\sigma = (\pi/(6)^{1/2})b$ for eight different mountain ranges representing over 600 estimates of extreme runout. The data imply (see Fig. 4) that $\mu_G/\sigma = 1.58$ ($R^2 = 0.86$) from ordinary least squares analysis with $p < 0.001$ or essentially that μ_G and σ are proportional (a constant added to the analysis is not significant). The model below is constructed to provide interpretations of μ_G/σ and σ in terms of snow climate and terrain for different mountain ranges.

Exponential crossing model

Consider a Poisson process to interpret variations of the ratio μ_G/σ in terms of a frequency index to enable frequency and terrain interpretation of Gumbel parameters for the different ranges. First consider a Poisson crossing process as a function of runout ratio (x^*) whereby avalanches reach or exceed a location x^* in the runout zone with temporal Poisson arrival of events. The problem is to estimate the frequency of events crossing a position x^* in the runout zone. The runout ratio is represented by X, with a value x^* denoting a reference position to be *crossed*. It is assumed that avalanche events have runout as a function of x^* with an exponential distribution, where $X = 0$ is a reference position (e.g., the β point). Hamre and McCarty (1996) display data in support of the exponential assumption.

The runout distance $X(t)$ is taken as a stationary random process (e.g., see Fig. 2a) and its time derivative as $\dot{X}(t)$. The probability that $X(t)$ and $\dot{X}(t)$ will be found within $(x, x + dx)$ and $(\dot{x}, \dot{x} + d\dot{x})$ is (e.g., Sólnes 1997) $f(x, \dot{x})\, dx\, d\dot{x} = f(x, \dot{x})\, \dot{x}\, d\dot{x}\, dt \simeq \Delta t\, f(x, \dot{x})\, \dot{x}\, d\dot{x}$, where $f(x, \dot{x})$ is the probability density function for $X(t)$ and $\dot{X}(t)$. The time interval Δt is finite but is defined such that the probability of two crossings occurring simultaneously is small. For a given value of x^* to be exceeded, $X(t)$ must reach x^* and the speed $\dot{x}$ must be positive. The exceedance probability (assumed small) of crossing between t and $t + \Delta t$ is then given by evaluating the probability density function at x^* over the short interval Δt for positive values of $\dot{x}$. The result is (e.g., Rice 1954; Sólnes 1997, p. 161)

$$[6] \qquad P(X > x^*) \simeq \Delta t \int_0^{\infty} f(x^*, \dot{x})\dot{x}\, d\dot{x}$$

To derive eq. [6], Δt is assumed small mathematically during the time integration. However, for rare events, Δt is not physically small except to preserve the assumption of rare, independent events.

It is now assumed that $f(x, \dot{x})$ may be represented as an exponential distribution:

$$[7] \qquad f(x, \dot{x}) = \frac{1}{ac} \exp\left[-\frac{x}{a} - \frac{\dot{x}}{c}\right]$$

with $x > 0$, $\dot{x} > 0$, and $a,c > 0$. Substitution in eq. [6] gives

$$[8] \qquad P(X > x^*) = \Delta t \frac{c}{a} \exp\left(-\frac{x^*}{a}\right) = \Delta t \lambda$$

where a and c are related to the variances D_x and $D_{\dot{x}}$ by $a = (D_x)^{1/2}$ and $c = (D_{\dot{x}})^{1/2}$. The crossing frequency (crossings per unit time) at x^* is then

$$[9] \qquad \lambda = \frac{1}{T_0} \exp\left(-\frac{x^*}{a}\right)$$

where $1/T_0 = n/\Delta t$, and n is the total average number of crossings at reference position $x^* = 0$ over the time interval Δt. The parameter n is taken to be a *frequency index* for a given mountain range.

Note that $f(\dot{x})$ could have been taken as a spike such as the Dirac delta function $\delta(\dot{x} - d)$ (or a sum of δ functions as in Fig. 2a) and the form of eq. [9] would not change if x and $\dot{x}$ are statistically independent. For example, if

$$f(\dot{x}) = \frac{1}{C_0} \sum_{k=1}^{n} C_k \delta(\dot{x} - d_k)$$

then with n spikes in time interval Δt, with strengths C_k, c in eq. [8] may be replaced by the expectation of $\dot{x}$, $E(\dot{x})$:

$$\frac{\sum_{k=1}^{n} C_k d_k}{C_0} \quad \text{with} \quad C_0 = \sum_{k=1}^{n} C_k$$

In general, the form of eq. [9] does not change given the assumption of statistical independence [$f(x, \dot{x}) = g(x) f(\dot{x})$]: $c \rightarrow E(\dot{x})$.

Now consider the arrival events to be Poisson distributed such that the probability x^* is exceeded depends only on the duration of the time interval Δt, no events occur simultaneously, and all events are statistically independent. The probability mass function is then

$$[10] \qquad P_k(\lambda \Delta t) = \frac{(\lambda \Delta t)^k \exp(-\lambda \Delta t)}{k!}$$

When the distance x^* increases, the time interval between Poisson crossings increases and the probability of one crossing is given asymptotically by

$$[11] \qquad P_1 = \lambda \Delta t$$

The probability that x* will not be crossed during Δt is (from 10 with $k = 0$)

$$[12] \qquad P_0 = \exp(-\lambda \Delta t)$$

Equation [12] is analogous to an asymptotic form of eq. [2]. Substituting the form of $\lambda \Delta t$ from the crossing model gives P_0 as the probability of no crossings from eq. [12]:

$$[13] \qquad P_0 = \exp -\left[\exp -\left(\frac{x^* - a\ln(n)}{a}\right)\right]$$

Equation [13] represents a Gumbel distribution where P_0 is equivalent to the nonexceedance probability evaluated at x^* when $\lambda \Delta t$ is small (long runout distances). From eq. [13], x^* is expressed in quantiles:

$$[14] \qquad x^* = a\ln(n) + a[-\ln(-\ln(P_0))]$$

From eq. [14], the location parameter is [$a \ln(n)$] and the scale parameter is a. The mean value of the distribution is

$$[15] \qquad \mu_G = E(x^*) = a\ln(n) + \gamma a$$

and the standard deviation is

$$[16] \qquad \sigma = \frac{\pi}{\sqrt{6}} a$$

and, therefore, for the assumptions, the ratio is

$$[17] \qquad \frac{\mu_G}{\sigma} = \frac{\ln(n) + \gamma}{\pi / \sqrt{6}}$$

The ratio (mean to standard deviation) then depends only on an index of frequency of events $\ln(n)$ for the assumptions made. The implications of eq. [17] on the implied frequency index are explored by computing the mean and standard deviation from Gumbel parameters for eight mountain ranges and equating the results to eq. [17]. Equation [17] contains the prediction that the ratio increases with the frequency index, implying higher overall frequency as the ratio increases. Table 1 gives frequency index values (n) for the different mountain ranges (from eq. [17]) and the distribution mean values found by least squares fits for lines through the avalanche runout data as described in Appendix 1. Figure 4 shows variations in the ratio for eight different mountain ranges, and Fig. 5 shows Gumbel plots from calculated parameters for the Canadian Rockies, Norway, and the Sierra Nevada: low-, medium-, and high-frequency ranges. In these plots, the location parameter increases with an increase in frequency (generally higher in the maritime ranges: see Table 1), but the spread of the distribution, a, does not depend on frequency according to the model.

Fig. 5. Gumbel probability density functions (PDF) for runout ratio for three mountain ranges. The spread (scale parameter) of the distributions depends on terrain steepness and the ratio of mean to standard deviation depends on frequency. In order of increasing avalanche frequency the ranges rank as follows: Canadian Rockies, Norway, and Sierra Nevada. The distribution parameters u and b were determined by least squares fits to extreme runout data (see Appendix 1).

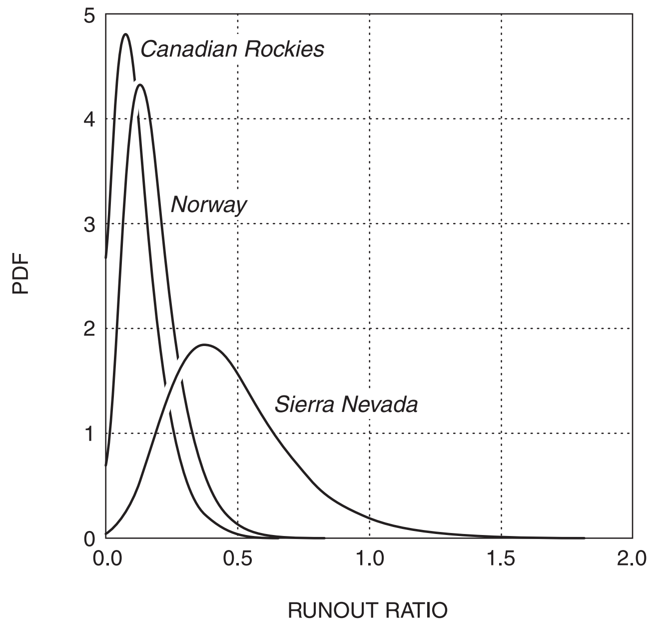


Table 2 shows the relationship between the spread parameter (a) and the steepness of terrain (represented by the mean value of α, where $\tan \alpha$ is the mean slope of a path and α is defined in Fig. 1) for the seven mountain ranges. Rank correlation of the mean of α and a for the seven mountain ranges in Table 2 is –0.93 with significance $p = 0.01$, i.e., a highly significant relationship. The model, in combination with the runout data, suggests that gentle terrain has a higher spread of runout distances and that avalanche frequency and overall features of terrain steepness (mean of α or tangent of mean of α) are separate issues.

The exponential Poisson crossing model fits the description of Gumbel distribution characteristics derived from worldwide experience: the location parameter increases with the spread of the distributions (i.e., in the model) but there is no significant relation between the scale parameter and the predicted avalanche frequency in the model as implied by the extreme-value data (see Table 1). The spread parameter, a, depends mostly on terrain, whereas the avalanche

Table 2. Relation between steepness (mean value of α) and spread parameter *a*.

Mountain range	Mean α	*a*	*N*	Remarks
Canadian Rockies	27.8	0.08	125	Continental
Norway	29.4	0.09	127	Maritime
British Columbia coast	26.8	0.09	31	Maritime
Alaska	25.4	0.11	52	Maritime
Iceland	23.6	0.13	45	Maritime – low frequency
Colorado	22.1	0.20	130	Continental
Sierra Nevada	20.1	0.20	90	Maritime

frequency is more closely related to snow supply, so no strong relation is expected between the two.

Gaussian crossing model

If $f(x)$ and $f(\dot{x})$ are statistically independent Gaussian processes, instead of exponential, then

$$[18] \qquad f(x) = \frac{2}{\sqrt{2\pi D_x}} \exp\left(-\frac{x^2}{2D_x}\right) \qquad x > 0$$

with a similar expression for $\dot{x}$ by replacing x with $\dot{x}$.

Given eq. [18], repeating the derivations above gives the result derived by Rice (1954) and Sólnes (1997, p. 162):

$$[19] \qquad \lambda = \frac{n}{\Delta t} \exp\left[-\frac{(x^*)^2}{2D_x}\right]$$

The result is a Gumbel distribution with $(x^*)^2$ instead of x^* as the runout variable:

$$[20] \qquad (x^*)^2 = 2D_x \ln(n) + 2D_x[-\ln(-\ln(P_0))]$$

From eq. [20], the ratio of the mean to the standard deviation of the distribution is the same as that given by eq. [17]. Therefore, the general frequency interpretation of the ratio is unchanged if the distribution variable is $(x^*)^2$ instead of x^*.

Extensive runout data show that the runout ratio x^* rather than $(x^*)^2$ provides a better fit to the data, so the exponential model is favoured over the Gaussian model for the frequency interpretation here. Calculations of Gumbel parameters using the Gaussian model also change the frequency rankings slightly compared with those in Table 1. For five mountain ranges, the calculations similar to Table 1 give the following: Sierra Nevada (n = 1.91, R^2 = 0.92), Alaska (n = 1.80, R^2 = 0.91), Colorado (n = 1.47, R^2 = 0.81), Norway (n = 1.00, R^2 = 0.94), and Canadian Rockies (n = 1.00, R^2 = 0.84). From a physical and mathematical point of view, then, I favour the exponential rather than the Gaussian crossing model because it ranks frequency index predictions in the maritime and continental ranges closest to experience and, more importantly, provides the best fits to extreme runout data.

The standard errors at long runout distances derived from fits to the Gumbel distributions imply longer runout distances and have greater uncertainty either within a mountain range in general *or* between mountain ranges. For different ranges (e.g., Fig. 5), the mean of the Gumbel distribution $u + \gamma b$ is taken as the index of runout, where a higher mean implies longer runout (see Table 3), and the ratio of the mean to the standard deviation (μ_G/σ) is taken as the index of frequency (see Table 1), where a higher ratio implies a higher frequency index regardless of the choice of crossing model. These definitions imply ranges with the longest runout distances require both a high frequency and a high spread parameter (implying gentler terrain on average), with the spread parameter dominating (Table 2).

Table 3. Mean values of Gumbel distributions for the runout ratio RR ($\Delta x/X_\beta$).

Mountain range	Mean	*N*	Remarks
Sierra Nevada	0.49	90	Maritime
Colorado	0.41	130	Continental
Northwest Iceland	0.26	25	Maritime
Alaska	0.25	52	Maritime
Norway	0.18	127	Maritime
British Columbia coast	0.16	31	Maritime
Eastern Iceland	0.14	20	Low frequency
Canadian Rockies	0.11	125	Continental

Note: The mean value is proposed as an index of runout.

If the distribution (eq. [6]) is generalized to include offset values α_0, $x \to x - \alpha_0$, such that it is not peaked at $x = 0$, then the form of the location parameter generalizes to $\alpha_0 + a\ln(n)$ and the scale parameter remains the same, so the form of the parameters matches that quoted by Sólnes (1997) as general properties of asymptotic extreme-value distributions: location parameter $\to \alpha_0 + \alpha_1 \ln(n)$, and scale parameter $\to \alpha_1$. Therefore, my choice of a distribution peaked at $x = 0$ affects the form of the extreme-value parameters but not the general conclusions made about frequency indexes and extreme runout based on the crossing model. A model peaked at α_0 implies that the ratio μ_G/σ is inversely proportional to the spread of the distribution for the exponential crossing model.

Discussion

The important points made in this paper are given below:

(1) The criticism that the topographic method applies to only one return period is addressed in this paper. A model has been produced to specify extreme runout in space and time in the runout zone to provide estimates of return period as a function of position in the runout zone. It is not necessary to assume that the extreme avalanche obeys a Gumbel distribution as done in this paper. However, the formalism requires the CDF of extreme runout for application. The other assumption is that avalanches arrive randomly according to a Poisson distribution (Föhn 1975, 1978; Smith and McClung 1997; McClung 1999) as rare, independent events.

(2) There is a consistent relation between Gumbel parameters from one mountain range to another. The ratio μ_G/σ appears to be relatively constant but variations in the ratio have been interpreted in terms of a frequency index, *n*, by applying a model which contains the assumptions that extreme runout for a mountain range follows a Gumbel distribution with arrival of avalanches according to a Poisson process. The model implies that the frequency index is higher (higher μ_G/σ ratio) for the maritime ranges than for continental ranges for cases studied thus far. This is reasonable because maritime ranges generally have a higher snow supply than continental ranges (McClung and Schaerer 1993) and, therefore, one might expect higher frequency on average for maritime ranges. This follows because avalanche frequency is strongly related to snow supply (Smith and McClung 1997).

(3) The mean value ($\mu_G = u + \gamma b$) of a Gumbel distribution is proposed as a measure of extreme runout for a mountain range: the higher the mean value for a mountain range, the longer are runout distances for that range versus a range with a lower mean. The spread or scale parameter *a* of the Gumbel distribution is related to overall terrain steepness: increasing with decreasing terrain steepness (mean value of α) according to the modelling in this paper.

(4) There is probably no general topographic model to predict the spatial variation of runout for all mountain ranges of the world. As shown in Table 1, the Gumbel distribution provides very good fits to extreme runout data for mountain ranges studied so far, and there is also evidence (see Appendix 2) that spatial extreme-value statistics are necessary for prediction of some extreme events. However, the model proposed for return-period prediction (e.g., eq. [4]) can be used with the CDF of extreme runout for the most suitable empirical distribution. The discussion in Appendix 2 shows that a Gaussian distribution fails in the case of Bleie, whereas the Gumbel distribution provides an adequate model. This result is very important, since most people using statistical models assume extreme runout follows a Gaussian distribution in practice. Thus, Bleie provides motivation for choice of the Gumbel distribution. Also, Bleie shows that the fit to a distribution is not the only criterion governing choice of a topographic model.

(5) One example (e.g., Bleie) does not provide enough validation for any model: further examples should be sought. Therefore, this paper should be viewed as a model presentation, with validation to follow. However, the empirical relationships (e.g., Gumbel distribution for runout) proposed here are based on extreme runout from more than 600 avalanche paths collected in eight different mountain ranges collected over more than 20 years. These extreme data collectively represent information from many thousands of individual avalanches. The other major assumption (Poisson arrival) is also physically reasonable and strongly backed by data (Smith and McClung 1997). Therefore, I believe the assumptions in this paper and the fundamental model of extremes and the consistency of the model results combine to make a very serious proposal as a practical tool.

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Appendix 1. Gumbel distribution applied to runout

A method for prescribing the CDF for extreme runout for a given mountain range based on Gumbel statistics was proposed by McClung and Lied (1987) and extended and expanded by McClung et al. (1989), McClung and Mears (1991), and Nixon and McClung (1993). See Lied and Bakkehøi (1980), McClung and Mears (1991), and Jóhannesson (1998) for descriptions of the data including errors. McClung and Mears showed the necessity of using data from a given mountain range to specify runout in that range, i.e., there are considerable differences between Gumbel parameters from one mountain range to another.

The runout parameter is the runout ratio (RR): the ratio of the horizontal reach Δx (extreme runout from the β point) to X_β, the horizontal reach from the start position to the β point (see Fig. 1). The idea is physically appealing, since a set of extreme values (runout) should follow an extreme-value distribution. In addition, the RR explicitly contains a length scale and implicitly accounts for slope of the terrain in the runout zone in terms of δ, the angle obtained by sighting from the extreme runout position (the α point) to the β point. In terms of α, β, and δ the runout ratio is

$$\text{[A1]} \quad \mathrm{RR} = \frac{\Delta x}{X_\beta} = \frac{\tan\beta - \tan\alpha}{\tan\alpha - \tan\delta}$$

Based on data from more than 600 different extreme runout measurements from eight different mountain ranges, the RR obeys a Gumbel distribution. McClung and Mears (1991) found very good fits to the Gumbel distribution, particularly at exceedance probabilities greater than 0.5, representing positions where applications are found. If RR $\equiv x$, then the CDF is given by

$$\text{[A2]} \quad F_X(x) = \exp - \left[\exp - \left(\frac{x-u}{b}\right)\right]$$

where u is the location parameter, and b is the scale parameter. In terms of quantiles, eq. [A2] is

$$\text{[A3]} \quad x = u + b[-\ln(-\ln(P)]$$

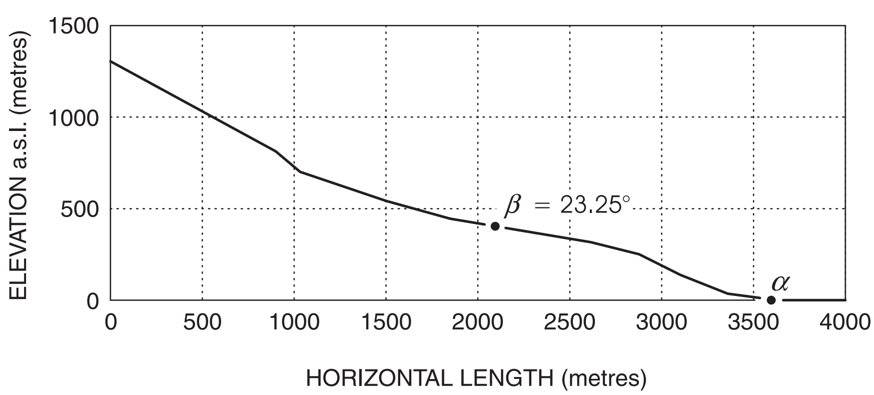
where P is the nonexceedance probability. In this paper, the constants u and b have been determined by a least squares fit to eq. [A3] by defining Hazen plotting positions in terms data x_i and the nonexceedance probability as $P_i = (i - 0.5)/N$, where i is the rank of the RR for avalanche path i, and N is the number of paths in the analysis.

Appendix 2: Calculation of the return period in the runout zone for Bleie, Norway

Here, estimates of return period are presented as a function of position in the runout zone for an avalanche path at Bleie, Norway, which has very long return periods associated with it according to the description given by Lied et al. (1998). Figure A1 shows the path profile. It was assumed that the end of the avalanche path has a total horizontal reach of 3600 m with vertical drop of 1300 m, the α angle is 19.9°, and the β angle is 23.25° as specified by Lied et al., who described avalanche occurrences for the avalanche path with annual occurrences reaching 500 m asl and below. Two avalanches have travelled to 300 m asl (one in 1776 to about 300 m asl) and another in 1994 reached the Bleie farms (3600 m horizontal reach) to yield an approximate return period of about 100 years at 300 m asl (approximately 2900 m reach).

For the model here, the Gumbel scale parameter depends mostly on terrain and the location parameter depends on frequency and terrain. The general form of the location parameter for a Gumbel distribution may be represented by u_n =

Fig. A1. Terrain profile for Bleie from Lied et al. (1998). The location at a horizontal reach of 3600 m (extreme runout) has a runout ratio of 0.714 with the β point taken at 2100 m. See Fig. 1 for the definition of runout ratio.



$\lambda_0 + \lambda_1 \ln(n)$, and the scale parameter is $b_n = \lambda_1$ (see, e.g., the crossing models introduced above or Sólnes 1997). For example, eq. [14] presents a Gumbel distribution for a reference position $x^* = 0$ (corresponding to the β point where $x^* = 0$). To extend the prediction to another reference position x_0, replacement $x^* \rightarrow x^* - x_0$ can be made, the exponential distribution (eq. [7]) peaks at $x^* - x_0$, and the location parameter becomes $x_0 + a \ln(n)$. This preserves the scale parameter dependence, and the location parameter can therefore be increased by the runout ratio of the new reference position (0.38 or 2900 m horizontal reach in this case). The value of μ_0 is then chosen as the Poisson parameter appropriate to the new reference position with longer return period. Keylock et al. (1998) employed a similar strategy for choice of reference position in their simulation model.

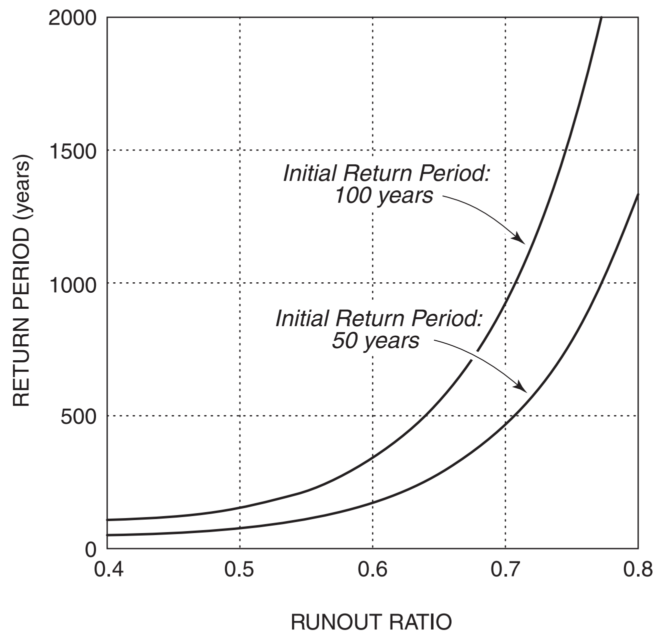
Given the data at the site, a reference position has been selected at 300 m asl (700 m uphill from the Bleie farms): $\mu_0 = 0.01$. However, since the return period at this position can probably not be estimated by better than a factor of two, calculations have also been made for the more conservative return period of 50 years at the reference position. Given these values for μ_0 (0.01 and 0.02) and the Gumbel distribution of extreme runout for Norway, plots are shown of the return period T in the runout zone versus the runout ratio X (= $\Delta x/X_\beta$) in Fig. A2. With X_β = 2100 m, the position at the end of the avalanche path (3600 m) is x = 0.714 in Fig. A1, a location near the Bleie farms. From these plots, the best estimate of the return period for Bleie is about 1000 years, with a more conservative estimate of about 500 years. If the return period at the 300 m asl position was as short as 50 years, then it is highly likely that more avalanches reaching or exceeding that point would have been recorded (if the return period there is 50 years, then for 200 years of observations the probability of observing at least three events is 0.76, whereas only two events have been recorded).

The written history of Bleie dates back to 1293 A.D. Lied et al. (1998) suggest that no earlier major avalanches have hit the farm for the last 1000 years. Lied et al. estimate that an appropriate return period for the location of the farms is in the range of 800–1000 years based on the local topographic and climate conditions and the historical records from Bleie.

The calculations were repeated by taking the reference position at the β point ($x = 0$) with approximately annual occurrences ($\mu = 1$) according to the descriptions given by Lied et al. (1998). These calculations gave a return period at the end of the paths (3600 m horizontal reach) of about 700 years, which is in good agreement with the calculations for the 300 m asl reference point.

The results above do not follow from the assumption that avalanche runout obeys Gaussian statistics using the α angle as a measure of runout with β as the predictor variable as is commonly done (e.g., Jóhannesson 1998). A least squares regression using Norwegian data gives the following values: $\alpha = 0.90\beta$, standard error = 1.86°, 127 avalanche paths with $R^2 = 0.87$ (see Jóhannesson 1998 for more information and rationale about the method and equation). According to Lied et al. (1998) the value of α (19.9°) for the 1994 avalanche at Bleie is only about half a standard deviation from the mean

Fig. A2. Return period in the runout zone for Bleie with initial return periods of 100 and 50 years taken at a horizontal reach of 2900 m. The return period at 3600 m horizontal reach (runout ratio = 0.714 and maximum runout position) is 1000 and 500 years, respectively.



for Norwegian data. I calculated the exceedance probability from the α,β regression model at about 0.29 (or the 1:3.5 avalanche runout) for α = 19.9° estimated at Bleie. This compares with 0.0015 (or the 1:700 avalanche runout) for the Gumbel RR method using Norwegian data.

If β is abandoned as a reference and statistics for α are used alone as an index of runout, again realistic return periods are not predicted. Lied et al. (1998) state that six of 206 avalanche runout distances have α less than 21°, which implies that Bleie is something like the 1:35 avalanche runout. The differences in these approaches are dramatic for the Bleie event and they are linked to the strong runout zone slope angle dependence (δ) of the α,β model the probability distributions used (Gumbel or Gaussian) and the methods used to calculate the probabilities from field data.

The case history of Bleie is extremely important because it clearly highlights the differences in predictions of the extreme-value approach and the α,β regression approach. To achieve an exceedance probability close to 0.0015 (as implied for the Bleie farms for the RR approach) using the α,β regression approach with Norwegian data, the implied α angle would have to be near 15°, which is equivalent to a horizontal reach near 4700 m, 1100 m farther than that observed. Similarly, if the exceedance probability calculated from the α,β model (0.29 at 3600 m horizontal reach) is used in eq. [2], the implied return period at the Bleie farms is about 4 years if the return period at the β point is about 1 year as described by Lied et al. (1998). If the α angle is used alone as a measure of runout, the exceedance probability for Bleie is near 0.03 to yield an estimated return period at Bleie of about 30 years using eq. [2] for annual return ($\mu_0 = 1$) at the β point.